International Journal of Research in Advent Technology, Vol.6, No.11, November 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

A New Form of $\mathcal{J}_{Nmg^{\#}}$-Closed Sets with Respect to Nano Ideal Spaces

[1]B. Theanmalar, [2]S. Nagarani and [3]A. Pandi
[1]*Department of Mathematics,*
Madurai Kamaraj University College,
Madurai-625 002, Tamil , India.
e-mail : theanmalars@gmail.com
[2]*Department of Mathematics,*
N. M. S. S. V. N. College, Nagamalai,
Madurai, Tamil Nadu, India.
e-mail : murugadossnagarani@yahoo.com
[3]*Ettunali Pudur, Vidathakulam(Post),*
Thirumangalam(TK), Madurai District-625 706,
Tamil Nadu, India.
e-mail : pandi2085@yahoo.com.

Abstract- Recently, V. BA.Vijeyrani and A.Pandi [21] introduced the notion of $Nmg^{\#}$-closed sets and properties of $Nmg^{\#}$-closed sets were investigated. In this paper, we introduce the notion of $\mathcal{J}_{Nmg^{\#}}$-closed sets and obtain the certain families of subsets between nano $*$-closed sets and $\mathcal{J}_{Nmg^{\#}}$-closed sets in an nano ideal minimal structure spaces.

Keyword- Nmg^{*}-closed set, n$\mathcal{J}_{g^*}$-closed set, nano $*$-closed set and $\mathcal{J}_{Nmg^{\#}}$-closed set.

1. INTRODUCTION

In 1970, Levine [8] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. M. K. R. S. Veera kumar [19] introduced a new class of sets, namely $g^{\#}$-closed sets in topological spaces. Veerakumar [20] introduced the notion of g^{*}-closed sets in topological spaces.

By combining a topological space (X, τ) and an ideal $\mathcal{J}$ on (X, τ), Ravi et al. [16] introduced the notion of $\mathcal{J}_{g^*}$-closed sets and investigated the properties of $\mathcal{J}_{g^*}$-closed sets.

An ideal $\mathcal{J}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

(1) A$\in \mathcal{J}$ and B$\subseteq$A$\Rightarrow$B$\in \mathcal{J}$ and

(2) A$\in \mathcal{J}$ and B$\in \mathcal{J}$ $\Rightarrow$A$\cup$B$\in \mathcal{J}$.

Given a topological space (X, τ) with an ideal $\mathcal{J}$ on X and if $\wp$(X) is the set of all subsets of X, a set operator $(.)^*$: $\wp$(X)$\rightarrow$ $\wp$(X), called a local function [5] of A with respect to τ and $\mathcal{J}$ is defined as follows: for A$\subseteq$X, $A^*(\mathcal{J},\tau)$={x$\in$X : U$\cap$A$\notin \mathcal{J}$ for every U$\in \tau$(x)} where τ(x)={U$\in \tau$: x$\in$U}. We will make use of the basic facts about the local functions ([4], Theorem 2.3) without mentioning it explicitly. A Kuratowski closure operator cl^*(.) for a topology $\tau^*(\mathcal{J},\tau)$, called the $*$-topology, finer than τ is defined by $cl^*(A)=A\cup A^*(\mathcal{J},\tau)$ [18]. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{J},\tau)$ and τ^* for $\tau^*(\mathcal{J},\tau)$. If $\mathcal{J}$ is an ideal on X, then (X, τ, $\mathcal{J}$) is called an ideal topological space. A subset A of an ideal topological space (X, τ, $\mathcal{J}$) is $*$-closed [4] if $A^* \subseteq$A. The interior of a subset A in (X, $\tau^*(\mathcal{J})$) is denoted by int^*(A).

Pandi et.al [9] introduced the notion of Nmg^{*}-closed sets and properties of Nmg^{*}-closed sets were investigated.

Recently, V.BA.Vijeyrani and A.Pandi [21] introduced the notion of $Nmg^{\#}$-closed sets are obtain the unified characterizations for certain families of subsets between nano closed sets and $Nmg^{\#}$-closed sets were investigated.

In this paper, we introduce the notion of $\mathcal{J}_{Nmg^{\#}}$-closed sets in nano ideal minimal structure spaces. we obtain some basic properties of $\mathcal{J}_{Nmg^{\#}}$-closed sets. we define several new subsets in nano ideal minimal structure spaces which lie between nano $*$-closed sets and $\mathcal{J}_{Nmg^{\#}}$-closed sets.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, n-$cl(A)$ and n-$int(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1 *[12] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x): R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 *[6] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 *[6] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:*

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 *[6] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5 *[6] If $(U, \tau_R(X))$ is a nano topological space with respect to U and if $A \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of A and it is denoted by n-$int(A)$.*

That is, n-$int(A)$ is the largest nano open subset of A.

The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by n-$cl(A)$.

That is, n-$cl(A)$ is the smallest nano closed set containing A.

Definition 2.6 *[3] An ideal $\mathcal{I}$ on a nano topological space (X, τ) is a nonempty collection of subsets of X which satisfies*

(1) A$\in \mathcal{I}$ and B $\subseteq$ A $\Rightarrow$ B $\in \mathcal{I}$ and
(2) A$\in \mathcal{I}$ and B $\in \mathcal{I}$ $\Rightarrow$ A $\cup$ B $\in \mathcal{I}$.

Definition 2.7 *[7] A nano topological space $(U, \tau_R(X))$ with an ideal $\mathcal{I}$ on U is called a nano ideal topological space or nano ideal space and denoted as $(U, \tau_R(X), \mathcal{I})$.*

Definition 2.8 *[7] Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal topological space. If $\wp(X)$ is the set of all subsets of*

U, a set operator $(.)^*$: $\wp(U) \rightarrow \wp(U)$ is called the nano local function of $\mathcal{I}$ on U with respect to $\mathcal{I}$ on $\tau_R(X)$ is defined as $A_n^* = \{x \in U: U \cap A \notin \mathcal{I}$, for every $U \in \tau_R(X)\}$ and is denoted by A_n^*, where nano closure operator is defined as n-cl *(A) = $A \cup A_n^*$.

Result 2.9 *[7] Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal topological space and A and B be subsets of U. Then*

1. A ⊆ B $\Rightarrow A_n^* \subseteq B_n^*$

2. A_n^* = n-cl(A_n^*) ⊆ n-cl(A) (A_n^* is a n-closed subset of n-cl(A)),

3. $(A_n^*)_n^* \subseteq A_n^*$,

4. $(A \cup B)_n^* = A_n^* \cup B_n^*$,

5. For every nano open set V $\Rightarrow V \cap A_n^* = V \cap (V \cap A)_n^* \subseteq (V \cap A)_n^*$,

6. $J \in \mathcal{I} \Rightarrow (A \cup J)_n^* = A_n^* = (A - J)_n^*$

Result 2.10 *[7] Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal topological space and A be a subset of U, If $A \subseteq A_n^*$, then A_n^* = n-cl(A_n^*) = n-cl(A) = n-cl *.*

Theorem 2.11 *[11] In a space $(U, \tau_R(X), \mathcal{I})$, if A and B are subsets of U, then the following results are true for the set operator n-cl *.*

1. A ⊆ n-cl *(A),

2. n-cl *(∅) = ∅ and n-cl *(U) = U,

3. If A ⊂ B, then n-cl *(A) ⊆ n-cl *(B),

4. n-cl *(A) ∪ n-cl *(B) = n-cl *(A ∪ B),

5. n-cl *(n-cl *(A)) = n-cl *(A).

Definition 2.12 *A subset A of a nano topological space $(U, \tau_R(X))$ is called*

1. nano regular-open [6] if A = n-int(n-cl(A)).

2. nano π-open [1] if the finite union of nano regular-open sets.

The family of nano regular-open (resp. nano π-open) sets is denoted by $NRO(U, \tau_R(X))$ (resp. $N\pi$O(U, $\tau_R(X)$)).

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.13 *[2] A subset A of a nano topological space $(U, \tau_R(X))$ is called nano g-closed if n-$cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.*

The complements of nano g-closed set is nano g-open set.

The family of all nano g-open sets of U is denoted by $NgO(U, \tau_R(X))$.

Definition 2.14 *A subset A of a nano ideal topological space $(U, \tau_R(X), \mathcal{I})$ is called*

1. nano $*$-closed [10] if $A_n^* \subseteq$ A.

2. nano regular-$\mathcal{I}$-open(briefly, n$\mathcal{I}$-regular-open) [7] if A = n-int(n-cl *(A)).

3. nano π-$\mathcal{I}$-open(briefly, π-n$\mathcal{I}$-open) [14] if the finite union of nano regular-$\mathcal{I}$-open sets.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.15 *A subset A of a nano ideal topological space $(U, \tau_R(X), \mathcal{I})$ is called;*

1. nano $\mathcal{I}$-g-closed(briefly, n$\mathcal{I}_g$-closed) [10] if A_n^* ⊆ G whenever $A \subseteq G$ and G is nano open.

2. nano $\mathcal{I}$-g^*-closed(briefly, n$\mathcal{I}_{g^*}$-closed) [15] if A_n^* ⊆ G whenever $A \subseteq G$ and G is nano g-open.

The complements of the above mentioned sets is called their respective open sets.

Definition 2.16 *[13] A subfamily m_X of the power set $\wp(X)$ of a nonempty set X is called a minimal structure (briefly m-structure) on X if $\phi \in m_X$ and $U \in m_X$.*

By (X, m_X), we denote a nonempty set X with a minimal structure m_X on X and call it a m-space. Each member of m_X is said to be m_X-open (briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (briefly m-closed).

Definition 2.17 *[9] A nano subfamily Nm_U of the power set $\wp(U)$ of a nonempty set U is called a nano* *minimal structure (briefly Nm-structure) on U if $\phi \in Nm_U$ and $U \in Nm_U$.*

By (U, Nm_U), we denote a nonempty set U with a nano minimal structure Nm_U on U and call it a nano m-space(briefly Nm-space). Each member of Nm_U is said to be nano Nm_U-open (briefly Nm-open) and the complement of an nano Nm_U-open set is said to be nano Nm_U-closed (briefly Nm-closed).

Definition 2.18 *[9] A nano topological space $(U, \tau_R(X))$ with a nano minimal structure Nm_U on U is called a nano minimal structure space $(U, \tau_R(X), Nm_U)$.*

Definition 2.19 *[9] Let (U, Nm_U) be an Nm-space. For a subset A of U, the Nm_u-closure of A and the Nm_u-interior of A are defined in as follows:*

1. $Nm_U\text{-}cl(A) = \cap\{F : A \subseteq F, F^c \in Nm_U\}$.

2. $Nm_U\text{-}int(A) = \cup\{V : V \subseteq A, V \in Nm_U\}$.

Definition 2.20 *[9] Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be*

1. nano minimal generalized closed(briefly Nmg-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nm_U-open.

2. nano minimal generalized open(briefly Nmg-open) if its complement is called Nmg-closed.

The family of all Nmg-open sets in U is an Nm-structure on U and denoted by $NmgO(U, \tau_R(X), Nm_U)$.

Definition 2.21 *[21] Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be*

1. nano minimal generalized closed(briefly Nmg^*-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nmg-open.

2. nano minimal generalized open(briefly Nmg^*-open) if its complement is Nmg^*-closed.

The family of all Nmg^*-open sets in U is an Nm-structure on U and denoted by $Nmg^*O(U, \tau_R(X), Nm_U)$.

Definition 2.22 *[21] Let $(U, Nmg^*O(U, \tau_R(X), Nm_U))$ be a nano minimal structure space. For a subset A of U, the Nmg^*-closure of A and the Nmg^*-interior of A are defined as follows:*

1. Nmg^*-cl(A) = $\cap\{F : A \subseteq F, U - F \in Nmg^*O(U, \tau_R(X), Nm_U)\}$,

2. Nmg^*-int(A) = $\cup\{V : V \subseteq A, V \in Nmg^*O(U, \tau_R(X), Nm_U)\}$.

3. $\mathcal{I}_{Nmg^\#}$-CLOSED SETS

We obtain several basic properties of $\mathcal{I}_{Nmg^\#}$-closed sets.

Definition 3.1 *A subset A of a nano ideal topological space $(U, \tau_R(X), \mathcal{I})$ is called;*

1. nano $\mathcal{I}$-rg-closed(briefly, n$\mathcal{I}_{rg}$-closed) if $A_n^* \subseteq G$ whenever $A \subseteq G$ and G is nano regular-open.

2. nano $\mathcal{I}$-πg-closed(briefly, n$\mathcal{I}_{\pi g}$-closed) if $A_n^* \subseteq G$ whenever $A \subseteq G$ and G is nano π-open.

The complements of the above mentioned sets is called their respective open sets.

Definition 3.2 *A nano ideal topological space $(U, \tau_R(X), \mathcal{I})$ with a nano minimal structure Nm_U on U is called a nano ideal minimal structure space $(U, \tau_R(X), \mathcal{I}, Nm_U)$.*

Definition 3.3 $(U, \tau_R(X), \mathcal{I}, Nm_U)$ *be a nano ideal minimal structure space. A subset A of U is said to be*

1. nano $\mathcal{I}_{mg^\#}$-closed(briefly, $\mathcal{I}_{Nmg^\#}$-closed) if $A_n^* \subseteq G$ whenever $A \subseteq G$ and G is Nmg^*-open.

2. $\mathcal{I}_{Nmg^\#}$-open if its complement is $\mathcal{I}_{Nmg^\#}$-closed.

Remark 3.4 *Let $(U, \tau_R(X), \mathcal{I}, Nm_U)$ be an nano ideal minimal structure space and A a subset of U. If $Nmg^*\ O\ (U, \tau_R(X), \mathcal{I}, Nm_U) = Ng\ O\ (U, \tau_R(X))$ (resp. $\tau_R(X)$, $N\pi\ O(U, \tau_R(X))$, $NRO(U, \tau_R(X))$) and A is nano $\mathcal{I}_{g^*}$-closed, then A is said to be $\mathcal{I}_{Nmg^\#}$-closed (resp. n $\mathcal{I}_g$-closed, n $\mathcal{I}_{\pi g}$-closed, n$\mathcal{I}_{rg}$-closed).*

Proposition 3.5 *Every nano $*$-closed set is*

$n\mathcal{I}_{g^*}$-closed.

Proof. It is obvious.

Example 3.6 *Let U = {a, b, c} with $U/R = \{\{b\},\{a,c\}\}$ and $X = \{c\}$. Then the nano topology $\tau_R(X) = \{\emptyset, U, \{a, c\}\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$ and $Nm_U = \{\emptyset, U\}$. Then nano $*$-closed sets are $\emptyset$, U, {b}, {c}, {b, c} and $n\mathcal{I}_{g^*}$-closed sets are $\emptyset$, U, {b}, {c}, {a, b}, {b, c}. It is clear that {a, b} is $n\mathcal{I}_{g^*}$-closed set but it is not nano $*$-closed.*

Proposition 3.7 *Let $Nmg^*O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$. Then every $n\mathcal{I}_{g^*}$-closed set is $\mathcal{I}_{Nmg^\#}$-closed but not conversely.*

Proof. Suppose that A is an $n\mathcal{I}_{g^*}$-closed set. Let A $\subseteq$ G and G $\in$ Nmg^*O(U, $\tau_R(X)$, Nm_U). Since Nmg^*O(U, $\tau_R(X)$, Nm_U) $\subseteq$ NgO(U, $\tau_R(X)$), A_n^* $\subseteq$ G and hence A is $\mathcal{I}_{Nmg^\#}$-closed.

Example 3.8 *Let U = {a, b, c, d} with $U/R = \{\{a\},\{b,c,d\}\}$ and $X = \{a,b\}$. Then the nano topology $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c, d\}\}$, $\mathcal{I} = \{\emptyset\}$ and $Nm_U = \{\emptyset, U, \{a, b\}, \{a, c\}, \{b, d\}\}$. Then $n\mathcal{I}_{g^*}$-closed sets are $\emptyset$, U, {a}, {b, c, d} and $\mathcal{I}_{Nmg^\#}$-closed sets are $\emptyset$, U, {a}, {a, d}, {b, c}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}. It is clear that {a, d} is $\mathcal{I}_{Nmg^\#}$-closed set but it is not $n\mathcal{I}_{g^*}$-closed.*

Remark 3.9 *Let $Nmg^*O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$. Then we have the following implications for the subsets stated above.*

nano $*$-closed $\rightarrow$ $n\mathcal{I}_{g^*}$-closed $\rightarrow \mathcal{I}_{Nmg^\#}$

Corollary 3.10 *If A and B are $\mathcal{I}_{Nmg^\#}$-closed sets in $(U, \tau_R(X), \mathcal{I}, Nm_U)$, then A $\cup$ B is $\mathcal{I}_{Nmg^\#}$-closed.*

Proof. Let A $\cup$ B $\subseteq$ G where G is Nmg^*-open. Then A $\subseteq$ G and B $\subseteq$ G. Since A and B are $\mathcal{I}_{Nmg^\#}$-closed, then A_n^* $\subseteq$ G and B_n^* $\subseteq$ G and so A_n^* $\cup$ B_n^* $\subseteq$ G. By Result 2.9, $(A \cup B)_n^* = A_n^* \cup B_n^*$. Hence A $\cup$ B is $\mathcal{I}_{Nmg^\#}$-closed.

Definition 3.11 *[21] An Nm-structure $Nmg^*O(U, \tau_R(X), Nm_U)$ on a nonempty set U is said to have property $\mathcal{J}_{\mathcal{N}}$ if the union of any family of subsets belonging to $Nmg^*O(U, \tau_R(X), Nm_U)$ belongs to $Nmg^*O(U, \tau_R(X))$.*

Example 3.12 *Let U = {a, b, c, d} with $U/R = \{\{a\},\{b,c,d\}\}$ and $X = \{a,b\}$. Then the nano topology $\tau_R(X) = \{\emptyset, U, \{a\}, \{b, c, d\}\}$ and $Nm_U = \{\emptyset, U, \{a, b\}, \{a, c\}, \{b, d\}\}$. Then Nmg^*-open sets are $\emptyset$, U, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {b, d} and {b, c, d}. It is shown that $Nmg^*O(U, \tau_R(X), Nm_U)$ does not have property $\mathcal{R}_{\mathcal{N}}$.*

Proposition 3.13 *Let $Nmg^*O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ and $Nmg^*O(U, \tau_R(X), Nm_U)$ have property $\mathcal{J}_{\mathcal{N}}$. If A is $\mathcal{I}_{Nmg^\#}$-closed set and B is nano $*$-closed set, then A $\cap$ B is $\mathcal{I}_{Nmg^\#}$-closed.*

Proof. Let A $\cap$ B $\subseteq$ G where G is Nmg^*-open. Then we have A $\subseteq$ G $\cup$(U$-$B). Since $\tau_R(X)$ $\subseteq$ Nmg^*O(U, $\tau_R(X)$, Nm_U) $\subseteq$ NgO(U, $\tau_R(X)$) and so G $\cup$(U$-$B) is Nmg^*-open. Since A is $\mathcal{I}_{Nmg^\#}$-closed, then A_n^* $\subseteq$ G $\cup$ (U$-$B) and hence A_n^* $\cap$ B$\subseteq$ G $\cap$ B $\subseteq$ G. By Result 2.9, $(A \cap B)_n^* \subseteq A_n^* \cap B_n^*$. Since $\tau_R(X) \subseteq \tau^*$, B is nano $*$-closed and B_n^* $\subseteq$ B. Therefore, we obtain $(A \cap B)_n^* \subseteq A_n^* \cap B_n^* \subseteq A_n^* \cap B_n^*$ $\subseteq$ G. This shows that A $\cap$ B is $\mathcal{I}_{Nmg^\#}$-closed.

Proposition 3.14 *If A is $\mathcal{I}_{Nmg^\#}$-closed and A $\subseteq$ B $\subseteq$ n-cl *(A), then B is $\mathcal{I}_{Nmg^\#}$-closed.*

Proof. Let B $\subseteq$ G where G is Nmg^*-open. Then A $\subseteq$ G and A is $\mathcal{I}_{Nmg^\#}$-closed. Therefore A_n^* $\subseteq$ G and B_n^* $\subseteq$ n-cl *(B) $\subseteq$ n-cl *(A) = A $\cup$ A_n^* $\subseteq$ G. Hence B is $\mathcal{I}_{Nmg^\#}$-closed.

Proposition 3.15 *A subset A of X is $\mathcal{I}_{Nmg^\#}$-open if and only if F $\subseteq$ n-int *(A) whenever F $\subseteq$ A and F is Nmg^*-closed.*

Proof. Suppose that A is $\mathcal{I}_{Nmg^\#}$-open. Let F $\subseteq$ A and F be Nmg^*-closed. Then U$-$A $\subseteq$ U$-$F and U$-$F is Nmg^*-open. Since U $-$ A is $\mathcal{I}_{Nmg^\#}$-closed, then $(U-A)_n^*$ $\subseteq$ U$-$F and U$-$n-int *(A) = n-cl *(U$-$A) = (U $-$ A) $\cup$ $(U-A)_n^*$ $\subseteq$ U $-$ F and hence F $\subseteq$ n-int *(A).
Conversely, let U$-$A $\subseteq$ G where G is Nmg^*-open. Then U$-$G $\subseteq$ A and U$-$G is Nmg^*-closed. By the hypothesis, we have U$-$G $\subseteq$ n-int *(A) and hence $(U-A)_n^*$ $\subseteq$ n-cl *(U$-$A) = U$-$n-int *(A) $\subseteq$ G. Therefore, U $-$ A is $\mathcal{I}_{Nmg^\#}$-closed and A is $\mathcal{I}_{Nmg^\#}$-open.

Corollary 3.16 *Let $Nmg^*O(U, \tau_R(X), Nm_U) \subseteq NgO(U, \tau_R(X))$ and $Nmg^*O(U, \tau_R(X), Nm_U)$ have property $\mathcal{J}_{\mathcal{N}}$. Then the following properties hold:*

1. Every nano $*$-open set is n$\mathcal{I}_{g^*}$-open and every $\mathcal{I}_{Nmg^\#}$-open set is $\mathcal{I}_{Nmg^\#}$-open,

2. If A and B are $\mathcal{I}_{Nmg^\#}$-open, then A $\cap$ B is $\mathcal{I}_{Nmg^\#}$-open,

3. If A is $\mathcal{I}_{Nmg^\#}$-open and B is nano $*$-open, then A $\cup$ B is $\mathcal{I}_{Nmg^\#}$-open,
4. If A is $\mathcal{I}_{Nmg^\#}$-open and n-int *(A) $\subseteq B \subseteq$ A, then B is $\mathcal{I}_{Nmg^\#}$-open.

Proof. This follows from Remark 3.9, Propositions 3.13 and 3.14 and Corollary 3.10.

4. CONCLUSION

In this paper, we define several new subsets in nano ideal minimal structure spaces which lie between nano $*$-closed sets and $\mathcal{I}_{Nmg^\#}$-closed sets are discussed. This shall be extended in the future Research with some applications.

REFERENCES

[1] Adinatha C. Upadhya, *On quasi nano p-normal spaces*, International Journal of Recent Scientific Research, 8(6)(2017), 17748-17751.
[2] K. Bhuvaneshwari and K. Mythili Gnanapriya, *Nano generalizesd closed sets*, International Journal of Scientific and Research Publications, 4(5)(2014), 1-3.
[3] T. R. Hamlett and D. Jonkovic, *Ideals in general topology*, Lecture notes in pure and Appl. Math., 123(1990), 115-125.
[4] D. Jankovic and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4)(1990), 295-310.
[5] K. Kuratowski, *Topology*, Vol. I. New York, Academic Press (1966).
[6] M. Lellis Thivagar and Carmel Richard, *On Nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention,1(1)(2013), 31-37.
[7] M. Lellis Thivagar and V. Sutha Devi, *New sort of operators in nano ideal topology*, Ultra Scientist Vol. 28(1)A (2016), 51-64.
[8] N. Levine, *Generalized closed sets in topology*, Rend. Circ Mat. Palermo (2), 19(1970), 89-96.
[9] A. Pandi, K. Banumathi, B. Theanmalar and V. BA. Vijeyrani, *Lies between nano closed sets and nano g^*-closed sets in nano minimal structure spaces*, International Journal of Research in Advent Technology, 6(10)(2018), 2568-2572.
[10] M. Parimala, S. Jafari and S. Murali, *Nano ideal generalized closed sets in nano ideal topological spaces*, Annales Univ. Sci. Budapest, 60(2017), 3-11.
[11] M. Parimala and S. Jafari, *On some new notions in nano ideal topological spaces*, International Balkan Journal of Mathematics (IBJM), 1 (3) (2018) 85-92.
[12] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5)(1982), 341-356.
[13] V. Popa and T. Noiri, *On M-continuous functions*, Anal. Univ. "Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor.(2), 18(23)(2000), 31–41.
[14] I. Rajasekaran and O. Nethaji, *Nano πg-closed sets with respect to an ideal*(communicated).
[15] V. Rajendran, P. Sathishmohan, K.Lavanya and R. Mangayarkarasi, *A new form of nano generalized closed sets in nano ideal topological spaces*, International Journal of Mathematics Trends and Technology (IJMTT), 57(6)(2018), 408-417.
[16] O. Ravi, M. Paranjothi, S. Murugesan and M. Meharin, *g^*-closed sets in ideal topological spaces*, South Asian Journal of Mathematics, 4(6) (2014), 252-264.
[17] P. Sulochana Devi and K. Bhuvaneswari , *On nano regular generalized and nano generalized regular closed sets in nano topological spaces*, International Journal of Engineering Trends and Technology (IJETT), 8(13)(2014), 386-390.
[18] R. Vaidyanathaswamy, Set topology, Chelsea Publishing Company (1946).
[19] M.K.R.S. Veera kumar, *$g^\#$-closed sets in topological spaces*, Mem. Fac. Sci. Kochi Uni. Ser. A. Math., 24(2003), 1-13.
[20] M.K.R.S. Veerakumar, *Between closed sets and g-closed sets*, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 21(2000), 1-19.
[21] V. BA. Vijeyrani and A. Pandi, *Note on $Nmg^\#$-closed sets in nano minimal structure spaces*, International Journal of Research in Advent Technology, 6(8)(2018), 1877-1882.